講義名　　　　解析力学

開講学期　　　後学期

単位数　　　　２

担当教員　　　フォレ・エティエンヌ　准教授

講義のねらい

The purpose of this course is to introduce practical techniques for the simulation and understanding of dynamical systems with special emphasis on accelerators. This course will consist of 15 lectures of 2 hours each (or a greater number of shorter lectures).

講義計画

1. Generic Properties of Dynamical Maps
2. Integration methods for ODE with accelerators in mind.
3. Fixed Point Classification, Jordan Canonical Forms and Floquet Theory
4. Nonlinear Perturbation and Floquet Theory using mostly Lie transforms

1)<u>Generic Properties:</u>

In order to illustrate generic properties, we will look at simple systems: the pendulum, the standard map, the logistic map, symbolic shift maps and other simple objects will permit us to explore, sometimes rigorously, the meaning of words such as integrability, resonances, chaos, (strange) attractors, etc … Lagrangians and Hamiltonians will be introduced. The goal is to present a geometrical picture of these phase space phenomena. Usage of computers and graphics will be important.

2)<u> Integration methods for ODE:</u>

In Part 2, the student learns how maps are constructed in typical accelerators. The basic approach is that of all modern tracking codes such as KEK's SAD code or CERN MAD-X. This part of the course is self-contained.

We will look at integration methods with a particular emphasis on symplectic integration: integration methods that preserve the Hamiltonian nature of the flow. We will examine the structure of the Hamiltonian used in accelerators and how one can perform piece-wise integration on the various magnets of the machine. We will also examine explicit and implicit schemes. We will introduce the computer concepts of automatic differentiation and operator overloading. If time permits, we will discuss some modern topics such as Butcher graph theory and the connection with symplectic Runge-Kutta.

3) Fixed Point Classification, Jordan Canonical Forms and Floquet Theory:

In Part 3, we will look at Hamiltonian and non-Hamiltonian perturbation theory. These theories are based, in modern parlance, on the classification of fixed points. Our attention with be concentrated on the closed orbit, the most important fixed point of accelerator physics. We will classify the linear motion around the closed orbit. We will study its decomposition into a semi-simple part and a nilpotent part (Jordan canonical forms). In an accelerator semi-simple systems are the back bone of the so-called Courant-Snyder: a Floquet Theory. However, in the case of a coasting beam, the linearized maps have also a nilpotent part mostly along the longitudinal dimension. The so-called momentum compaction is a parameter of the nilpotent part of the associated Jordan canonical form. We will also discuss limiting cases: when the nilpotent part is the result of a characteristic frequency approaching zero. This is at the core of various approximate theories such as the famous Sands synchrotron integrals.

4) Nonlinear Perturbation and Floquet Theory using mostly Lie transforms :

In Part 4, we will look at the nonlinear generalization of the operators described in Part 3. This process involves a linearization of the equation of motion at the expense of turning the system into an infinite dimensional one: once more a Floquet Theory emerges. We extend formally (leaving issues of convergence aside) the normal coordinates found in part 3 to the nonlinear system. In fact, perturbation theory is an attempt to locally distort phase space so that the full motion has the appearance of the cases discussed in Part 3. We will discuss issues of convergence which arise from such a bold attempt particularly in the light of the observed chaos and resonances present in the simple simulations of Part 1. This will also include a discussion of stochastic maps relevant to radiation (Sands integrals generalized). We will also compare our techniques with that of Quantum Mechanics: the perturbed harmonic oscillator. The Lie transforms techniques first introduced by Hori as well as generating functions methods will be discussed in detail.

Bibliography:

I will produce copies and pdf's of the necessary material. Perhaps the book which approximates my lectures in spirit is Michelotti's book. However it has nothing on actual integration methods. I recommend my pdf on the KEK site for an historical look at symplectic integration in accelerators (KEK Preprint 2005-107).

Here is a short list of books.

Accelerator Physics:

1. *Intermediate Classical Dynamics with Applications to Beam Physics*, Leo, Michelotti, Wiley Series. Covers Part 1,3 and 4 of my lectures in some details.
2. *Modern Map Methods for Particle Beam Physics*, Martin Berz, Academic Press. Covers Part 1,2,3 and 4 of my lectures in some details. As a lot of stuff on power series methods.
3. *Beam Dynamics: A New Attitude and Framework*, Etienne Forest, Harwood Academic Well, my book. The part on symplectic integration and perturbation theory will be treated again in the lecture.
4. S.Y. Lee's book and H. Weidemann's second book can also be consulted. The philosophy is completely different however, so beware.

Mathematics:

Dynamical Systems and Chaos:

1. *Differential Equations and Dynamical Systems*, Lawrence Perko, Springer Verlag. Contains the standard mathematical description.
2. *Differential Equation, Dynamical System and Linear Algebra*, Maurice W. Hirsh and Stephen Smale, Academic Press. Classic book which has a chapter in Jordan form and also describes several famous examples.!
3. *Chaos in Dynamical Systems*, Edward Ott, Cambridge Press. Nice little self-contained chapters on several topics: fractals, Lyapunov exponents,.KAM tori, etc…

Solving Equations on the Computer

1. *A first course in the Numerical Analysis of Differential Equations*, Arieh Iserles, Cambridge Texts in Applied Mathematics. Very nice book which balances well the need for rigour (it is a mathematics book) and the applied nature of numerical analysis. .
2. *Numerical Methods for Ordinary Differential Equations*, J.C. Butcher, Wiley. Another classic from the most famous expert on Runge-Kutta.
3. *Geometric Integration for Particle Accelerators*, E. Forest, KEK Preprint 2005-107. Very relevant to accelerators: invited review paper.